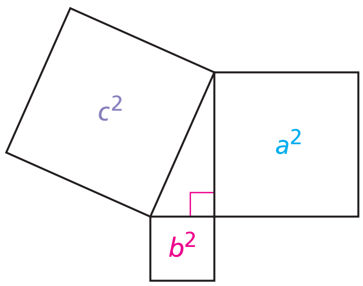
Investigation 4

Using the Pythagorean Theorem

In Investigation 3, you studied the Pythagorean Theorem, which states:

> The area of the square on the hypotenuse of a right triangle is equal to the sum of the areas of the squares on the legs.



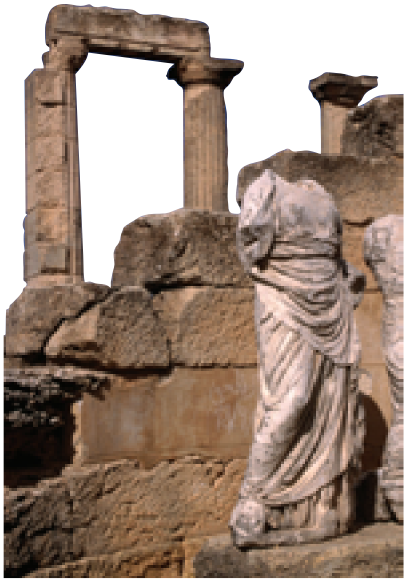
$a^2 + b^2 = c^2$

In this investigation, you will explore some interesting applications of the Pythagorean Theorem.

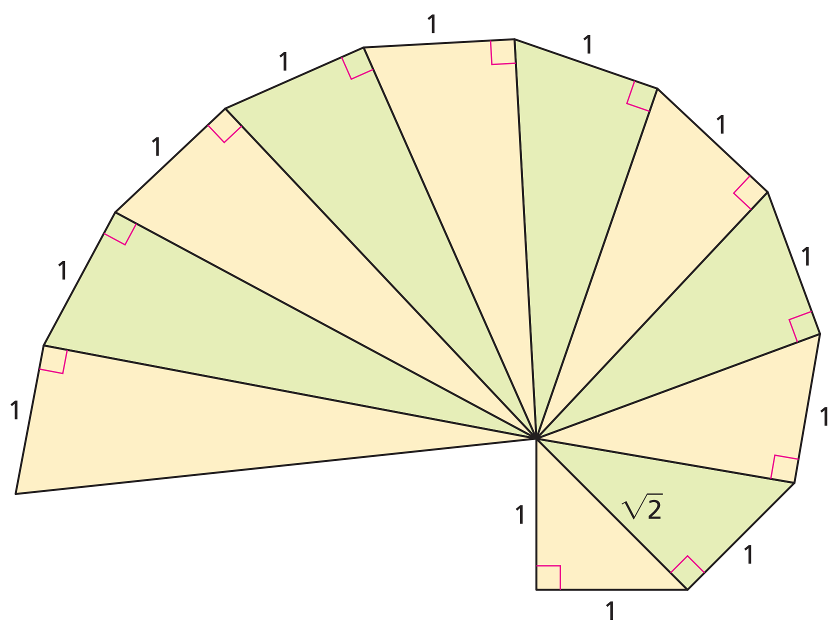
4.1 Analyzing The Wheel of Theodorus

The diagram on the next page is named for its creator, Theodorus of Cyrene (sy ree nee), a former Greek colony. Theodorus was a Pythagorean.

The Wheel of Theodorus begins with a triangle with legs 1 unit long and winds around counterclockwise. Each triangle is drawn using the hypotenuse of the previous triangle as one leg and a segment of length 1 unit as the other leg. To make the Wheel of Theodorus, you need only know how to draw right angles and segments 1 unit long.



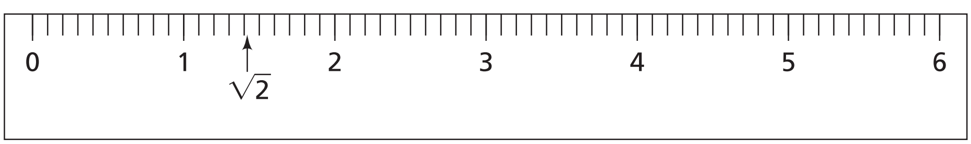
Wheel of Theodorus



Problem 4.1 Analyzing the Wheel of Theodorus

A. Use the Pythagorean Theorem to find the length of each hypotenuse in the Wheel of Theodorus. On a copy of the wheel, label each hypotenuse with its length. Use the $\sqrt{\ }$ symbol to express lengths that are not whole numbers.

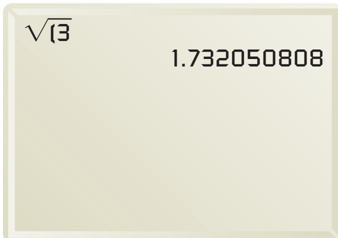
B. Use a cut-out copy of the ruler below to measure each hypotenuse on the wheel. Label the place on the ruler that represents the length of each hypotenuse. For example, the first hypotenuse length would be marked like this:

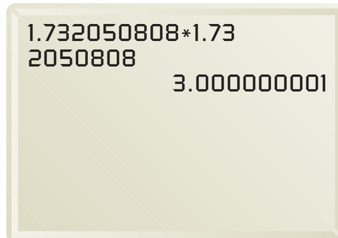


C. For each hypotenuse length that is not a whole number:

1. Give the two consecutive whole numbers the length is between. For example, $\sqrt{2}$ is between 1 and 2.
2. Use your ruler to find two decimal numbers (to the tenths place) the length is between. For example $\sqrt{2}$ is between 1.4 and 1.5.
3. Use your calculator to estimate the value of each length and compare the result to the approximations you found in part (2).

D. Odakota uses his calculator to find $\sqrt{3}$. He gets 1.732050808. Geeta says this must be wrong because when she multiplies 1.732050808 by 1.732050808, she gets 3.000000001. Why do these students disagree?





ACE **Homework starts on page 53.**

Did You Know?

Some decimals, such as 0.5 and 0.3125, *terminate*. They have a limited number of digits. Other decimals, such as 0.3333 . . . and 0.181818 . . . , have a repeating pattern of digits that never ends.

Terminating or repeating decimals are called **rational numbers** because they can be expressed as *ratios* of integers.

$$0.5 = \frac{1}{2} \qquad 0.3125 = \frac{5}{16} \qquad 0.3333\ldots = \frac{1}{3} \qquad 0.181818\ldots = \frac{2}{11}.$$

Some decimals neither terminate nor repeat. The decimal representation of the number π starts with the digits 3.14159265 . . . and goes forever without any repeating sequence of digits. Numbers with non-terminating and non-repeating decimal representations are called **irrational numbers.** They cannot be expressed as ratios of integers.

The number $\sqrt{2}$ is an irrational number. You had trouble finding an exact terminating or repeating decimal representation for $\sqrt{2}$ because such a representation does not exist. Other irrational numbers are $\sqrt{3}$, $\sqrt{5}$, and $\sqrt{11}$. In fact, $\sqrt{n}$ is an irrational number for any value of n that is not a square number.

The set of irrational and rational numbers is called the set of **real numbers.** An amazing fact about irrational numbers is that there is an infinite number of them between any two fractions!